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Reliable H_∞ Control for Discrete Networked Control Systems with Probabilistic Faults

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Abstract: This paper concerns the robust H_∞ control design for wireless networked control systems (NCSs) with probabilistic sensors or actuators faults, measurements distortion, random delay, packet dropout and system uncertainties. The faults of each sensors or actuators happen in a random way, which is governed by an individual random variable satisfying a certain probabilistic distribution over the interval $[0, \theta_l](l = 1, 2, \theta_l > 1)$. By introducing some random variables, new type of wireless NCS fault model is proposed. The merit of the proposed fault model lies in its generalization and reality, which can cover some existing fault models as special cases. By using Lyapunov functional method and linear matrix inequality technology, sufficient conditions are obtained for the robustly mean square stable (RMSS) of the wireless NCSs with an H_∞ norm bound γ. A simulation example is given to demonstrate the efficiency and application of the proposed method.

Key Words: Probabilistic failures, Robustly mean square stable, Networked control systems

1 Introduction

Networked control systems (NCSs), in which the information among distributed sensors, controllers and actuators exchanges through communication network, have received considerable attention in recent years [6, 19, 20, 5, 22, 14, 23, 13]. Based on the connection types of the communication network, the NCSs can be clarified into wire NCSs, wireless NCSs and hybrid NCSs. Among them, wireless NCSs are becoming dominant because of fully mobile operation, flexible installation, rapid development and less maintenance costs [1, 11, 3, 13, 19, 10, 4, 2, 12]. However, building a NCSs over wireless is more challenging due to the mobile of the nodes, the varying routing, signal-attenuation and so on, which can induce many challenging issues, such as random delay, packet loss, actuators/sensors faults and measurement distortion. Even though this is true for wire NCSs, however, it is much more pronounced in wireless NCSs due to limited spectrum and power, time-varying channel gains and interference.

Random delay and packet loss in the wireless networked control systems have caused some consideration in some references [21, 9]. In [21], a continuous wireless NCSs model is built and the problems of random delay and data quantization are taken into consideration. In [9], data losses and their effect on wireless networked system stability/performance is studied using a simple communication scheme and a stochastic 2-state Markov network model. As is known, the distributed sensors and actuators in the wireless NCSs are in different environments, which is affected by aging, temperature, disturbances, power, electromagnetic interference and some other disturbance. In this regard, it can be seen that the sensors/actuators work in different condition and the temporarily sensors/actuators failure (or partial failure) and measurement distortion are unavoidable, which is an important resource of instability and system performance degradation as well as the random delay and packet loss. Therefore, a very imminent need is to design the reliable controllers that are robust to the probabilistic sensors and actuators faults, measurement distortion, random delay, packet dropout and parameter uncertainties. However, it has not caused enough consideration and still need further investigation, which is the motivate of the present study.

This paper investigates the robust H_∞ control design for unreliable networked control systems. The distributed sensors or actuators may have temporary fault, the failure rate of each sensor or actuator is governed by a random variable. Under the cases of probabilistic sensor or actuator fault, measurements distortion, random delay and packet dropout, a new stochastic fault model is proposed. The purpose of this study is to design a reliable controller such that the robust mean square stability (RMSS) of the NCSs can be guaranteed for given H_∞ performance index γ. A numerical example is given to show the effectiveness of the proposed design procedures.

2 System Model Description

Consider the uncertain discrete-time linear model of the plant as follows

$$x_{k+1} = (A + \Delta A_k)x_k + (B + \Delta B_k)\,u_k + B_\omega \omega_k \quad (1)$$
$$z_k = Cx_k + D\omega_k \quad (2)$$

where $x_k \in \mathbb{R}^n, u_k \in \mathbb{R}^m$ and ω_k are respectively the state vector, control vector and disturbance input, $z(k)$ is the controlled output vector. A, B are matrices with compatible dimensions, ΔA_k and ΔB_k are parameter uncertainties which satisfy

$$\begin{bmatrix} \Delta A_k & \Delta B_k \end{bmatrix} = HF(k) \begin{bmatrix} E_1 & E_2 \end{bmatrix}, \quad (3)$$

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where H, E_1 and E_2 are constant matrices with appropriate dimensions, $F(k)$ is unknown time-varying matrix which satisfies $\|F(k)\| \leq 1$.

Assumption 1 *In this paper, the distributed sensors, controllers and actuators are assumed to be connected through network, d_k^{sc} and d_k^{ca} denote the transmission delay of the packet at times k from sensor to controller and from controller to actuator respectively, and their sum d_k is bounded by d_M, that is, $d_k^{sc} + d_k^{ca} = d_k \leq d_M$.*

Under Assumption 1, the controller u_k can be designed in the form of

$$u_k = Kx_{k-d_k} \tag{4}$$

where x_{k-d_k} is the abbreviation for $x(k-d_k)$. Furthermore, if we further consider the unreliable sensors and actuators, we have the following assumption.

Assumption 2 *The sensors or actuators' faults occur in a random way, the probabilistic failure of each sensor or actuator is governed by an individual random variable satisfying certain probabilistic distribution and taking values in an interval $[0, \theta_l](l = 1, 2, \theta_l > 1)$.*

Under Assumption 2, considering the probabilistic sensors and actuators faults, the controller (4) can be further described as

$$u_k = \Xi_2 K \Xi_1 x_{k-d_k} \tag{5}$$

where $\Xi_1 = diag\{\Xi_{11}, \Xi_{12}, \cdots, \Xi_{1n}\}$ with $\Xi_{1i}(i = 1, 2, \cdots, n)$ being n unrelated random variables taking values on the interval $[0, \theta_1]$, where $\theta_1 \geq 1$, the mathematical expectation and variance of $\Xi_{1i}(i = 1, 2, \cdots, m)$ are α_i and $\breve{\alpha}_i^2$. $\Xi_2 = diag\{\Xi_{21}, \Xi_{22}, \cdots, \Xi_{2m}\}$ with $\Xi_{2i}(i = 1, 2, \cdots, m)$ being m unrelated random variables taking values on the interval $[0, \theta_2]$, where $\theta_2 \geq 1$, the mathematical expectation and variance of $\Xi_{2i}(i = 1, 2, \cdots, m)$ are β_i and $\breve{\beta}_i^2$.

Remark 1 *Using the random variable to describe the sensor faults (also called missing measurement or packet loss) has also been considered in [16, 17, 15, 8]. In the above mentioned references, a random variables γ_k taking values in $\{0, 1\}$ is utilized, which can only represent completely failure or completely normal of the sensor. In [7], the authors introduced a general γ_k^i taking values in the interval $[0, 1]$, for $0 < \gamma_k^i < 1$, it means partial failure. However, in [16, 17, 15, 7], all the sensors are assumed to have identical failure characteristics, which is improved in [8, 18], wherein a set of new random variables are utilized, each sensor has individual failure rate.*

Remark 2 *As is known to all, when the sensors or actuators have faults, the output measurements from sensor or controller may be different from the real measurements, which is called measurements distortion. The measurements distortion may result performance degradation and even instability of the system. However, which has been omitted by most of the researchers.*

Remark 3 *In this paper, the probabilistic sensors or actuators faults are considered as well as the measurements distortion and networked delay. When $\Xi_{1i} = 0$(or $\Xi_{2j} = 0$), it means complete failure of the ith sensor (or jth actuator) or packet loss happening from sensor to controller (or from controller to actuator). When $\Xi_{1i} = 1$(or $\Xi_{2j} = 1$), it means the ith sensor (or jth actuator) is in a good work condition. When $\Xi_{1i} \in (0, 1)$ (or $\Xi_{2j} \in (0, 1)$), it means partial failure of the ith sensor (or jth actuator) or measurements distortion with the case of output measurement smaller than the real measurement. When $\Xi_{1i} \in (1, \theta_1)$ (or $\Xi_{2j} \in (1, \theta_2)$), it means measurements distortion with the case of output measurement larger than the real measurement.*

Firstly, assuming $\Delta A_k = 0$ and $\Delta B_k = 0$, substituting the reliable controller (5) into system (1), the probabilistic fault model of the NCSs is obtained as

$$\begin{aligned} x_k &= Ax_k + B\Xi_2 K \Xi_1 x_{k-d_k} + B_\omega \omega_k && (6)\\ &= Ax_k + B\bar{\Xi}_2 K \bar{\Xi}_1 x_{k-d_k} + B\left[\bar{\Xi}_2 K\left(\Xi_1 - \bar{\Xi}_1\right)\right. \\ &\quad + \left(\Xi_2 - \bar{\Xi}_2\right) K \bar{\Xi}_1 \\ &\quad \left. + \left(\Xi_2 - \bar{\Xi}_2\right) K \left(\Xi_1 - \bar{\Xi}_1\right)\right] x_{k-d_k} \\ &= A_{F1}\zeta_k + B_F x_{k-d_k} + B_\omega \omega_k \\ z_k &= Cx_k + D\omega_k && (7)\\ x_k &= \phi_k, k = -d_M, -d_M + 1, \cdots, -1, 0 && (8) \end{aligned}$$

where $\phi(k)$ is the initial condition of the state.

$$\begin{aligned} A_{F1} &= \left[\begin{array}{cccc} A & B\bar{\Xi}_2 K \bar{\Xi}_1 & 0 & B_\omega \end{array}\right] && (9)\\ B_F &= B\bar{\Xi}_2 K \Delta\Xi_1 + B\Delta\Xi_2 K \bar{\Xi}_1 \\ &\quad + B\Delta\Xi_2 K \Delta\Xi_1 \\ \zeta_k^T &= \left[\begin{array}{cccc} x_k^T & x_{k-d_k}^T & x_{k-d_M}^T & \omega_k^T \end{array}\right] \\ \bar{\Xi}_1 &= \mathcal{E}\{\Xi_1\} = \sum_{i=1}^{n} \alpha_i \Theta_1^i, \\ \bar{\Xi}_2 &= \mathcal{E}\{\Xi_2\} = \sum_{i=1}^{m} \beta_i \Theta_2^i, \\ \Theta_1^i &= diag\{\underbrace{0, \cdots, 0}_{i-1}, 1, \underbrace{0, \cdots, 0}_{n-i}\}, \\ \Theta_2^j &= diag\{\underbrace{0, \cdots, 0}_{j-1}, 1, \underbrace{0, \cdots, 0}_{m-j}\} \\ \Delta\Xi_1 &= \left(\Xi_1 - \bar{\Xi}_1\right), \Delta\Xi_2 = \left(\Xi_2 - \bar{\Xi}_2\right) \end{aligned}$$

where The purpose of this paper is to design the reliable H_∞ controller for the robust mean square stability (RMSS) of the NCSs (7)-(8) under the cases of probabilistic sensors and actuators faults, measurements distortion, random delay, packet losses and system norm bounded system uncertainties. A definition is proposed first.

Definition 1 *System (6)-(8) is said to RMSS with an H_∞ norm bound γ if the following hold:*

(i). System (6)-(8) with $w_k = 0$ is RMSS, that is, there exists a scalar $c > 0$ such that

$$\mathcal{E}\left\{\sum_{k=0}^{\infty} \|x_k\|^2\right\} \leq c\mathcal{E}\left\{\|\phi_k\|\right\}^2$$

(ii). Under zero initial condition, the controlled output $z(k)$ satisfies

$$\mathcal{E}\left\{\sum_{k=0}^{\infty} \|z_k\|^2\right\} \leq \gamma^2 \mathcal{E}\left\{\sum_{k=0}^{\infty} \|w_k\|^2\right\}$$

where $w_k \in \mathcal{L}_2 = \left\{ w_k : \mathcal{E}\left\{ \sum_{k=0}^{\infty} \|w_k\|^2 \right\} < \infty \right\}$.

3 Main Results

Based on the Lyapunov-Krasovskii functional method, we can obtain the following result.

Theorem 1 *System (6)-(8) is said to be RMSS if there exist matrices $P > 0$, $Q > 0$, $R > 0$, N, M with appropriate dimensions, such that for $l = 1, 2$*

$$\begin{bmatrix} \Pi_{11} & * & * & * & * \\ \Pi_{21}^l & -R & * & * & * \\ \mathcal{C} & 0 & -I & * & * \\ \Pi_{41} & 0 & 0 & \Pi_{44} & * \\ \Pi_{51} & 0 & 0 & 0 & \Pi_{55} \end{bmatrix} < 0 \qquad (10)$$

where A_{F1} is defined in (9) and

$$\begin{aligned} \Pi_{11} &= \Upsilon + \Gamma + \Gamma^T, \\ \Upsilon &= diag\{Q - P, 0, -Q, -\gamma^2 I\} \\ \Gamma &= \begin{bmatrix} -N & N - M & M & 0 \end{bmatrix} \\ \Pi_{21}^1 &= \sqrt{d_M} N^T, \Pi_{21}^2 = \sqrt{d_M} M^T, \\ \Pi_{41} &= \begin{bmatrix} A_{F1} \\ \sqrt{d_M} A_{F2} \end{bmatrix}, \Pi_{51} = \begin{bmatrix} \Pi \\ \sqrt{d_M} \Pi \end{bmatrix} \\ \Pi_{44} &= diag\{-P^{-1}, -R^{-1}\}, \\ \Pi_{55} &= diag\{-P^{-1}, \cdots, -R^{-1}, \cdots, -R^{-1}\} \\ \Pi^T &= \begin{bmatrix} \chi_1^T & \chi_2^T & \cdots & \chi_n^T \end{bmatrix} \\ \chi_j^T &= \begin{bmatrix} \mathcal{B}_{1j}^T & \mathcal{B}_{2j}^T & \cdots & \mathcal{B}_{mj}^T \end{bmatrix} \\ \mathcal{B}_{ij} &= \begin{bmatrix} 0 & \sqrt{v_{ij}} B \Theta_2^j K \Theta_1^i & 0 & 0 \end{bmatrix} \\ v_{ij} &= \breve{\alpha}_i^2 \beta_j + \alpha_i \breve{\beta}_j^2 + \breve{\alpha}_i^2 \breve{\beta}_j^2, \\ A_{F2} &= \begin{bmatrix} A - I & B \Xi_2 K \Xi_1 & 0 & B_\omega \end{bmatrix} \\ \mathcal{C} &= \begin{bmatrix} C & 0 & 0 & D \end{bmatrix} \\ \Pi_{66} &= diag\left\{-P^{-1}, \cdots, -P^{-1}\right\}_{mn \times mn}, \\ \Pi_{77} &= diag\left\{-R^{-1}, \cdots, -R^{-1}\right\}_{mn \times mn} \end{aligned}$$

Proof 1 *Construct the Lyapunov functional candidate as*

$$V_k = x_k^T P x_k + \sum_{i=k-d_M}^{k-1} x_i^T Q x_i + \sum_{i=-d_M}^{-1} \sum_{j=k+i}^{k-1} e_j^T R e_j \qquad (11)$$

where $P > 0, Q > 0$ and $R > 0$

$$e_k = x_{k+1} - x_k = A_{F2} \zeta_k + B_F x_{k-d_k}$$

Taking the forward difference for the Lyapunov functional and taking expectation on it, we obtain:

$$\begin{aligned} &\mathcal{E}\{\Delta V(k)\} \\ =\ &\mathcal{E}\left\{ \left(\zeta_k^T A_{F1}^T + x_{k-d_k}^T B_F^T\right) P \left(A_{F1}\zeta_k + B_F x_{k-d_k}\right) \right. \\ &-x_k^T P x_k + x_k^T Q x_k - x_{k-d_M}^T Q x_{k-d_M} + d_M e_k^T R e_k \\ &\left. - \sum_{i=k-d_M}^{k-1} e_i^T R e_i \right\} \end{aligned} \qquad (12)$$

Noting that $\mathcal{E}\{\Delta \Xi_1\} = diag\{0, 0, \cdots 0\}, \mathcal{E}\{\Delta \Xi_2\} = diag\{0, 0, \cdots 0\}$, using free weighing matrix method, we can obtain from (12) that

$$\begin{aligned} &\mathcal{E}\{\Delta V(k)\} \\ =\ &\mathcal{E}\left\{ \zeta_k^T A_{F1}^T P A_{F1} \zeta_k + x_{k-d_k}^T B_F^T P B_F x_{k-d_k} \right. \\ &+x_k^T (Q - P) x_k - x_{k-d_M}^T Q x_{k-d_M} + d_M e_k^T R e_k \\ &- \sum_{i=k-d_M}^{k-1} e_i^T R e_i - 2\zeta_k^T N \left[x_k - x_{k-d_k} - \sum_{i=k-d_k}^{k-1} e_i \right] \\ &-2\zeta_k^T M \left[x_{k-d_k} - x_{k-d_M} - \sum_{i=k-d_M}^{k--d_k-1} e_i \right] \\ &\left. +z_k^T z_k - \gamma^2 \omega_k^T \omega_k - z_k^T z_k + \gamma^2 \omega_k^T \omega_k \right\} \end{aligned} \qquad (13)$$

Noting that

$$z_k^T z_k = \zeta_k^T \mathcal{C}^T \mathcal{C} \zeta_k$$

Define $\digamma(i, j) = \left(B \Theta_2^j K \Theta_1^i\right)^T P \left(B \Theta_2^j K \Theta_1^i\right)$,

$$\begin{aligned} &\mathcal{E}\left\{ x_{k-d_k}^T B_F^T P B_F x_{k-d_k} \right\} \\ =\ &\mathcal{E}\left\{ \sum_{i=1}^{n} \sum_{j=1}^{m} v_{ij} x_{k-d_k}^T \digamma(i, j) x_{k-d_k} \right\} \\ =\ &\mathcal{E}\left\{ \sum_{i=1}^{n} \sum_{j=1}^{m} \zeta_k^T \mathcal{B}_{ij}^T P \mathcal{B}_{ij} \zeta_k^T \right\} \end{aligned} \qquad (14)$$

where v_{ij} and $\mathcal{B}_{ij}$ are defined in (10). Similarly

$$\begin{aligned} &\mathcal{E}\left\{ d_M e_k^T R e_k \right\} = \mathcal{E}\left\{ d_M \zeta_k^T A_{F2}^T R A_{F2} \zeta_k \right. \qquad (15) \\ &+ \left. \sum_{i=1}^{n} \sum_{j=1}^{m} d_M \zeta_k^T \mathcal{B}_{ij}^T R \mathcal{B}_{ij} \zeta_k^T \right\} \end{aligned}$$

Substituting (14)-(15) into (13), we can obtain

$$\begin{aligned} &\mathcal{E}\{\Delta V_k\} = \mathcal{E}\left\{ \zeta_k^T W \zeta_k \right. \\ &+ \left. 2\zeta_k^T N \sum_{i=k-d_k}^{k-1} e_i + 2\zeta_k^T M \sum_{i=k-d_M}^{k-d_k-1} e_i \right\} \\ &-\mathcal{E}\left\{ - \sum_{i=k-d_M}^{k-1} e_i^T R e_i z_k^T z_k - \gamma^2 \omega_k^T \omega_k \right\} \\ =\ &\mathcal{E}\left\{ \frac{1}{d_M} \sum_{i=k-d_k}^{k-1} \tilde{\zeta}_{k,i}^T \digamma_1 \tilde{\zeta}_{k,i} - z_k^T z_k \right. \\ &\left. + \frac{1}{d_M} \sum_{i=k-d_M}^{k-d_k-1} \tilde{\zeta}_{k,i}^T \digamma_2 \tilde{\zeta}_{k,i} + \gamma^2 \omega_k^T \omega_k \right\} \end{aligned} \qquad (16)$$

where

$$\begin{aligned} W &= \Upsilon + \Gamma + \Gamma^T + \mathcal{C}^T \mathcal{C} + A_{F1}^T P A_{F1} \\ &\quad + d_M A_{F2}^T R A_{F2} \\ &\quad + \sum_{i=1}^{n} \sum_{j=1}^{m} \mathcal{B}_{ij}^T (P + d_M R) \mathcal{B}_{ij} \\ \tilde{\zeta}_{k,i}^T &= \begin{bmatrix} \zeta_k^T & e_i^T \end{bmatrix}, \digamma_1 = \begin{bmatrix} W & d_M N \\ d_M N^T & -d_M R \end{bmatrix}, \\ \digamma_2 &= \begin{bmatrix} W & d_M M \\ d_M M^T & -d_M R \end{bmatrix} \end{aligned}$$

By Schur complements, it can be seen that $F_l < 0$ are equivalent to (10) for $l = 1, 2$, respectively. Therefore, if condition (10) is satisfied, we can conclude that

$$\mathcal{E}\left\{\Delta V_k\right\} \le -\lambda\mathcal{E}\left\{\zeta_k^T\zeta_k\right\} - \mathcal{E}\left\{z_k^T z_k - \gamma^2\omega_k^T\omega_k\right\} \quad (17)$$

where $\lambda = \min\left\{\lambda_{\min}\{W + d_M N^T R^{-1} N\}\right.$ $\left., \lambda_{\min}\{W + d_M M^T R^{-1} M\}\right\}$. We will first show the system (6)-(8) with $\omega_k = 0$ is RMSS. When $\omega_k = 0$, (17) implies

$$\mathcal{E}\left\{\Delta V_k\right\} \le -\lambda\mathcal{E}\left\{\zeta_k^T\zeta_k\right\} \le -\lambda\mathcal{E}\left\{x_k^T x_k\right\} \quad (18)$$

Summing up the above inequality from $k = 0$ to ∞ yields

$$\mathcal{E}\left\{\sum_{k=0}^{\infty} x_k^T x_k\right\} \le \frac{1}{\lambda}\mathcal{E}\left\{V(0)\right\} \quad (19)$$

From the construction of V_k, we can show that there exists a constant c such that

$$\mathcal{E}\left\{V(0)\right\} \le \lambda c \sup_{-d_M\le i\le 0}\mathcal{E}\left\{\phi_k^T\phi_k\right\} \quad (20)$$

Next, assuming that under the zero initial condition, we will show that system (6)-(8) has a prescribed level γ of H_∞ noise attenuation. In fact, notice that (17) implies

$$\mathcal{E}\left\{\Delta V_k\right\} \le -\mathcal{E}\left\{z_k^T z_k - \gamma^2\omega_k^T\omega_k\right\} \quad (21)$$

Summing both sides of (21) from 0 to ∞, we obtain

$$\begin{aligned}&\mathcal{E}\left\{\sum_{k=0}^{\infty} z_k^T z_k\right\} \quad (22)\\ \le\ & \gamma^2\mathcal{E}\left\{\sum_{k=0}^{\infty}\omega_k^T\omega_k\right\} + \mathcal{E}\left\{V(0)\right\} - \mathcal{E}\left\{V(\infty)\right\}\end{aligned}$$

From the construction of V_k and under zero initial condition, it is easy to see that $V(0) = 0$ and $V(\infty) > 0$. Thus, we obtain

$$\mathcal{E}\left\{\sum_{k=0}^{\infty} z_k^T z_k\right\} \le \gamma^2\mathcal{E}\left\{\sum_{k=0}^{\infty}\omega_k^T\omega_k\right\} \quad (23)$$

From Definition 1, the proof can be completed.

When considering the parameter uncertainties, system (6)-(8) can be rewritten as

$$\begin{aligned} x_k &= (A + \Delta A_k)x_k + (B + \Delta B_k)\,\Xi_2 K \Xi_1 x_{k-d_k} && (24)\\ z_k &= Cx_k + D\omega_k && (25)\\ x_k &= \phi_k, k = -d_M, -d_M+1, \cdots, -1, 0 && (26)\end{aligned}$$

Using a common method for parameter uncertainties, the RMSS conditions for system (24)-(26) can be derived based on Theorem 1.

Theorem 2 *System (24)-(26) is said to be RMSS if there exist matrices $P > 0$, $Q > 0$, $R > 0$, N, M with appropriate dimensions, such that for $l = 1, 2$*

$$\begin{bmatrix} \Pi_{11} & * & * & * & * & * & * \\ \Pi_{21}^l & -R & * & * & * & * & * \\ \mathcal{C} & 0 & -I & * & * & * & * \\ \Pi_{41} & 0 & 0 & \Pi_{44} & * & * & * \\ \Pi_{51} & 0 & 0 & 0 & \Pi_{55} & * & * \\ \Pi_{61} & 0 & 0 & \Pi_{64} & 0 & \Pi_{66} & * \\ \Pi_{71} & 0 & 0 & 0 & \Pi_{75} & 0 & \Pi_{66} \end{bmatrix} < 0 \quad (27)$$

where Π_{11}^l, $\Pi_{21}, \Pi_{41}, \Pi_{44}, \Pi_{51}$ and Π_{55} are as defined in (10),

$$\begin{aligned} \Pi_{61} &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ \varepsilon E_1 & \varepsilon E_2 & 0 & 0 \end{bmatrix}, \\ \Pi_{71} &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \varepsilon E_2 & 0 & 0 \end{bmatrix}, \\ \Pi_{64} &= \begin{bmatrix} H^T P & \sqrt{d_M} H^T R \\ 0 & 0 \end{bmatrix}, \\ \Pi_{75} &= \begin{bmatrix} \Pi_{751} & \Pi_{752} \end{bmatrix}, \\ \Pi_{751} &= \begin{bmatrix} H^T P & \cdots & H^T P \\ 0 & \cdots & 0 \end{bmatrix} \\ \Pi_{752} &= \begin{bmatrix} \sqrt{d_M} H^T R & \cdots & \sqrt{d_M} H^T R \\ 0 & \cdots & 0 \end{bmatrix} \\ \Pi_{66} &= diag\{-\varepsilon I, -\varepsilon I\}, \end{aligned}$$

In the existing references, the following methods are widely used to design the feedback gain K_j. For example, in [5, 20], by pre and post multiplying the stability conditions with $diag\{P^{-1}, ..., P^{-1}\}$ and defining some new parameters $X = P^{-1}, \tilde{Q} = XQX$, $Y = KX$. It should be noted that the obtained criteria are not strict LMIs because of the existence of $XR^{-1}X$, then by formulating it into a sequential optimization problem subject to LMI constraints, variable X and Y can be obtained, thus $K = YX^{-1}$ can be obtained. However, the above mentioned method can not be applied to the system with unreliable sensors: if we pre and post multiply $B_i\bar{\Pi}_2 K_j\bar{\Pi}_1$ with X, the stability criteria becomes a nonlinear one and can not be solved by LMI method. In this paper, a algorithm is proposed for Theorem 1 and 2 to solve out K_j.

Define new variables $\bar{P} = P^{-1}$ and $\bar{R} = R^{-1}$ and replace them in (27) (or (10)), the new obtained criteria are noted as (27)' (or (10)').

Algorithm 1 *(For Theorem 1 or Theorem 2)*

Given constants d_M and let c denote the maximum number of iterations.

(1) Find a feasible solution $\left\{P, \bar{P}, R, \bar{R}\right\}$ to LMIs (27)' (or (10)') and

$$\begin{bmatrix} P & I \\ I & \bar{P} \end{bmatrix} \ge 0 \quad \begin{bmatrix} R & I \\ I & \bar{R} \end{bmatrix} \ge 0 \quad (28)$$

If no feasible solution, EXIT. Else, set $k = 0$.

(2) Solve the following minimization problem:

$$\min\ \mathrm{tr}\left(P_k\bar{P} + \bar{P}_k P + R_k\bar{R} + \bar{R}_k R\right)$$

subject to LMIs (27)' (or (10)') and (28). (29)

(3) If (30) is satisfied for a sufficient small scalar $\varepsilon > 0$, output the feedback gain $K = YX^{-1}$. Otherwise, set $k = k + 1$. If $k < c$ (c denotes the maximum number of iterations), go to Step (1), otherwise, EXIT.

$$\left|\mathrm{tr}\left(P_k\bar{P} + \bar{P}_k P + R_k\bar{R} + \bar{R}_k R\right) - 4n\right| < \varepsilon \quad (30)$$

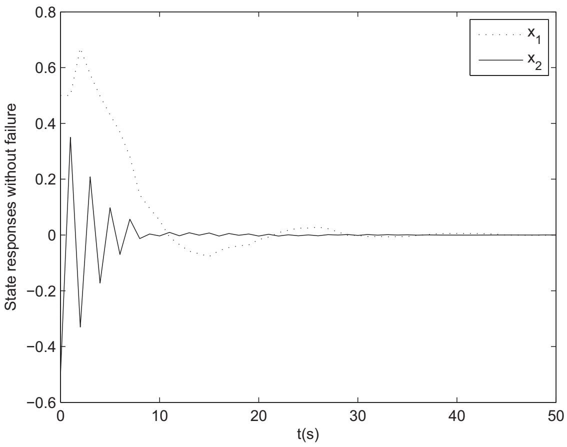

Fig. 1: The state responses for Case 1)

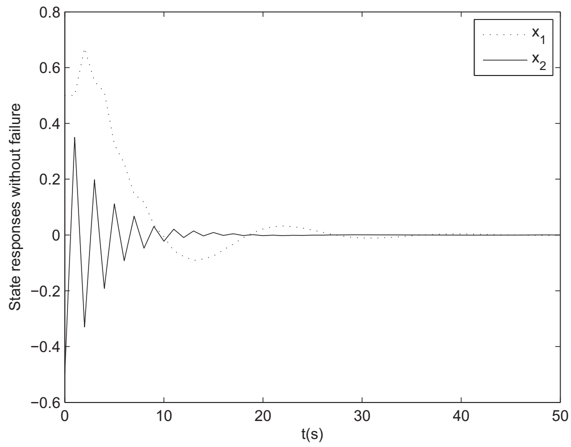

Fig. 2: The state responses for Case 2)

4 Numerical Example

Example 1 *Considering the following discrete-time system*

$$x_{k+1} = \begin{bmatrix} 1.2 & 0.2 \\ -0.1 & -0.8 \end{bmatrix} x_k + \begin{bmatrix} -0.5 & 0 \\ 0.2 & 0.9 \end{bmatrix} u_k + \begin{bmatrix} 0.5 \\ 0.1 \end{bmatrix} \omega_k \quad (31)$$

$$z_k = \begin{bmatrix} 0.1 & -0.1 \end{bmatrix} x_k + \omega_k \quad (32)$$

where

$$\omega_k = \begin{cases} 0.1, k \in [5, 8] \\ 0, otherwise \end{cases}$$

For system (31), to illustrate the efficiency and application of the proposed procedure, we consider the following 3 cases:

Case 1: System (31) with unreliable sensors and actuators and measurements distortion, the failure rates are $\bar{\Xi}_1 = diag\{0.5, 0.8\}, \bar{\Xi}_2 = diag\{0.8, 1.1\}, \breve{\alpha}_i = \breve{\beta}_j = 0.2$, *using Theorem 1 and Algorithm 1 with* $d_M = 2$, *we obtain* $\gamma_{\min} = 2.37$ *and the corresponding feedback gain is*

$$K = \begin{bmatrix} 1.7515 & 0.1499 \\ -0.1468 & 0.0241 \end{bmatrix} \quad (33)$$

For the initial state $x(0) = [0.5; -0.5]$, *the state responses are shown in Fig. 1. From Fig 1, it can be found that using the proposed method, the controller can stabilize the NCS under the cases of probabilistic sensor failures, actuators failures, measurements distortion, network-induced delay and packet dropout.*

Case 2: System (31) without unreliable issues, that is, $\bar{\Xi}_1 = diag\{1, 1\}, \bar{\Xi}_2 = diag\{1, 1\}, \breve{\alpha}_i = \breve{\beta}_j = 0$. *Using Theorem 1 and Algorithm 1 with* $d_M = 2$, *the minimum* H_∞ *performance index is obtained as* $\gamma_{\min} = 1.40$ *and controller feedback gain is*

$$K = \begin{bmatrix} 0.7503 & 0.0011 \\ -0.1373 & 0.0269 \end{bmatrix} \quad (34)$$

the state responses are shown in Fig. 2. From Fig 2, it can be found that the proposed method is also suitable for system without unreliable cases. Furthermore, from the obtained minimum H_∞ *performance index in Case 1 and Case 2, we can conclude that the happening of the sensors' or actuators' fault can cause performance degradation of the NCSs.*

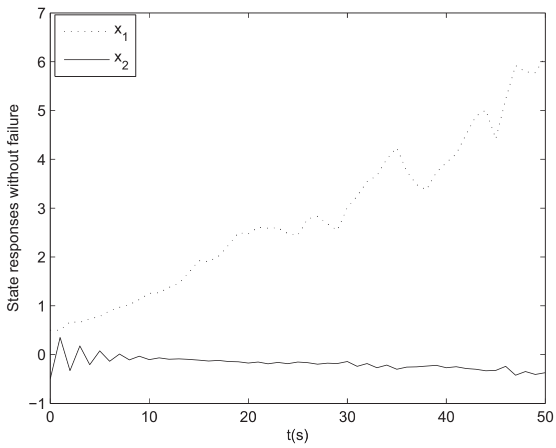

Fig. 3: The state responses for Case 3)

Case 3: The failure of sensors or actuators happen, however, we still use the controller designed for reliable systems. That is, for the failures rates $\bar{\Xi}_1 = diag\{0.5, 0.8\}, \bar{\Xi}_2 = diag\{0.8, 1.1\}, \breve{\alpha}_i = \breve{\beta}_j = 0.2$, *the controller feedback gain (34) is utilized, the state responses are shown in Fig. 3. From Fig 3, it can be found that the system becomes unstable using feedback gain (34) when failures happens, which demonstrates the necessity and importance of the reliable control design for NCS.*

For the system uncertainties (3) with parameters

$$H = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, E_1 = \begin{bmatrix} 0.04 & 0 \\ 0 & 0.03 \end{bmatrix},$$

$$E_2 = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}$$

Using Theorem 2 and Algorithm 1, for the parameters $\bar{\Xi}_1 = diag\{0.5, 0.8\}, \bar{\Xi}_2 = diag\{0.8, 1.1\}, \breve{\alpha}_i = \breve{\beta}_j = 0.2$ *and* $d_M = 2$, *the minimum* H_∞ *performance index is obtained as* $\gamma_{\min} = 6.30$.

5 Conclusion

This paper considers the reliable H_∞ control design for wireless networked control systems with norm bounded uncertainties, probabilistic sensors and actuators failures, measurements distortion, networked delays and packet losses. The faults of the sensors or actuators are assumed to be occurred in a random way, and their failure rate is governed by two sets of random variables. The merit of the proposed fault model and proposed method lies in its generalization and reality, which can cover some existing fault models as special cases. By using Lyapunov functional method, sufficient conditions for the RMSS of the wireless with H_∞ performance index γ is obtained. The given example shows the efficiency and application of the proposed method.

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